1 Common Evaluation Measures

- Recall
  A measure of the ability of a system to present all relevant items.
  \[
  \text{recall} = \frac{\text{number of relevant items retrieved}}{\text{number of relevant items in collection}}
  \]

- Precision.
  A measure of the ability of a system to present only relevant items.
  \[
  \text{precision} = \frac{\text{number of relevant items retrieved}}{\text{total number of items retrieved}}
  \]

Precision and recall are set-based measures. That is, they evaluate the quality of an unordered set of retrieved documents. To evaluate ranked lists, precision can be plotted against recall after each retrieved document as shown in the example below. To facilitate computing average performance over a set of topics—each with a different number of relevant documents—individual topic precision values are interpolated to a set of standard recall levels (0 to 1 in increments of .1). The particular rule used to interpolate precision at standard recall level \(i\) is to use the maximum precision obtained for the topic for any actual recall level greater than or equal to \(i\). Note that while precision is not defined at a recall of 0.0, this interpolation rule does define an interpolated value for recall level 0.0. In the example, the actual precision values are plotted with circles (and connected by a solid line) and the interpolated precision is shown with the dashed line.

Example: Assume a document collection has 20 documents, four of which are relevant to topic \(t\). Further assume a retrieval system ranks the relevant documents first, second, fourth, and fifteenth. The exact recall points are 0.25, 0.5, 0.75, and 1.0. Using the interpolation rule, the interpolated precision for all standard recall levels up to .5 is 1, the interpolated precision for recall levels .6 and .7 is .75, and the interpolated precision for recall levels .8 or greater is .27.
2 trec_eval Evaluation Report

Retrieval tasks whose results are a ranked list of documents can be evaluated by the trec_eval program. trec_eval was written by Chris Buckley. It is available from the TREC website at trec.nist.gov/trec_eval/. An evaluation report for a run evaluated by trec_eval is comprised of a header (containing the task and organization name), 3 tables, and 2 graphs as described below.

2.1 Tables

I. “Summary Statistics” Table

Table 1 is a sample “Summary Statistics” Table.

<table>
<thead>
<tr>
<th>Summary Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run</td>
</tr>
<tr>
<td>Cor7A1c1t-automatic, title</td>
</tr>
<tr>
<td>Number of Topics</td>
</tr>
<tr>
<td>50</td>
</tr>
<tr>
<td>Total number of documents over all topics</td>
</tr>
<tr>
<td>Retrieved: 50000</td>
</tr>
<tr>
<td>Relevant: 4674</td>
</tr>
<tr>
<td>Rel_ret: 2621</td>
</tr>
</tbody>
</table>

A. Run

A description of the run. It contains the run tag provided by the participant, and various details about the runs such as whether queries were constructed manually or automatically.

B. Number of Topics

Number of topics searched in this run (generally 50 topics are run for each task).

C. Total number of documents over all topics (the number of topics given in B).

i. Retrieved

Number of documents submitted to NIST. This is usually 50,000 (50 topics × 1000 documents), but is less when fewer than 1000 documents are retrieved per topic.

ii. Relevant

Total possible relevant documents within a given task and category.

iii. Rel_ret

Total number of relevant documents returned by a run over all the topics.

II. “Recall Level Precision Averages” Table.

Table 2 is a sample “Recall Level Precision Averages” Table.

A. Precision at 11 standard recall levels

The precision averages at 11 standard recall levels are used to compare the performance of different systems and as the input for plotting the recall-precision graph (see below). Each recall-precision average is computed by summing the interpolated precisions at the specified recall cutoff value (denoted by $\sum P_\lambda$ where $P_\lambda$ is the interpolated precision at recall level $\lambda$) and then dividing by the number of topics.

$$\frac{\sum_{\lambda=1}^{NUM} P_\lambda}{NUM} \quad \lambda = \{0.0, 0.1, 0.2, 0.3, \ldots, 1.0\}$$
Table 2: Sample “Recall Level Precision Averages” Table.

<table>
<thead>
<tr>
<th>Recall</th>
<th>Precision</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.6169</td>
</tr>
<tr>
<td>0.10</td>
<td>0.4517</td>
</tr>
<tr>
<td>0.20</td>
<td>0.3938</td>
</tr>
<tr>
<td>0.30</td>
<td>0.3243</td>
</tr>
<tr>
<td>0.40</td>
<td>0.2715</td>
</tr>
<tr>
<td>0.50</td>
<td>0.2224</td>
</tr>
<tr>
<td>0.60</td>
<td>0.1642</td>
</tr>
<tr>
<td>0.70</td>
<td>0.1342</td>
</tr>
<tr>
<td>0.80</td>
<td>0.0904</td>
</tr>
<tr>
<td>0.90</td>
<td>0.0472</td>
</tr>
<tr>
<td>1.00</td>
<td>0.0031</td>
</tr>
</tbody>
</table>

Average precision over all relevant docs
non-interpolated 0.2329

- Interpolating recall-precision
  Standard recall levels facilitate averaging and plotting retrieval results.

B. Average precision over all relevant documents, non-interpolated
This is a single-valued measure that reflects the performance over all relevant documents. It rewards systems that retrieve relevant documents quickly (highly ranked).
The measure is not an average of the precision at standard recall levels. Rather, it is the average of the precision value obtained after each relevant document is retrieved. (When a relevant document is not retrieved at all, its precision is assumed to be 0.) As an example, consider a query that has four relevant documents which are retrieved at ranks 1, 2, 4, and 7. The actual precision obtained when each relevant document is retrieved is 1, 1, 0.75, and 0.57, respectively, the mean of which is 0.83. Thus, the average precision over all relevant documents for this query is 0.83.

III. “Document Level Averages” Table
Table 3 is a sample “Document Level Averages” Table.

A. Precision at 9 document cutoff values
The precision computed after a given number of documents have been retrieved reflects the actual measured system performance as a user might see it. Each document precision average is computed by summing the precisions at the specified document cutoff value and dividing by the number of topics (50).

B. R-Precision
R-Precision is the precision after R documents have been retrieved, where R is the number of relevant documents for the topic. It de-emphasizes the exact ranking of the retrieved relevant documents, which can be particularly useful in TREC where there are large numbers of relevant documents.
The average R-Precision for a run is computed by taking the mean of the R-Precisions of the individual topics in the run. For example, assume a run consists of two topics, one with 50 relevant documents and another with 10 relevant documents. If the retrieval system returns 17 relevant documents in the top 50 documents for the first topic, and 7 relevant documents in the top 10 for the second topic, then the run’s R-Precision would be \( \frac{17}{50} + \frac{7}{10} \) or 0.52.
Table 3: Sample “Document Level Averages” Table.

<table>
<thead>
<tr>
<th>Document Level Averages</th>
<th>Precision</th>
</tr>
</thead>
<tbody>
<tr>
<td>At 5 docs</td>
<td>0.4280</td>
</tr>
<tr>
<td>At 10 docs</td>
<td>0.3960</td>
</tr>
<tr>
<td>At 15 docs</td>
<td>0.3493</td>
</tr>
<tr>
<td>At 20 docs</td>
<td>0.3370</td>
</tr>
<tr>
<td>At 30 docs</td>
<td>0.3100</td>
</tr>
<tr>
<td>At 100 docs</td>
<td>0.2106</td>
</tr>
<tr>
<td>At 200 docs</td>
<td>0.1544</td>
</tr>
<tr>
<td>At 500 docs</td>
<td>0.0875</td>
</tr>
<tr>
<td>At 1000 docs</td>
<td>0.0524</td>
</tr>
</tbody>
</table>

R-Precision (precision after R docs retrieved (where R is the number of relevant documents))

| Exact                  | 0.2564    |

2.2 Graphs

I. Recall-Precision Graph

Figure 1 is a sample Recall-Precision Graph.

Figure 1: Sample Recall-Precision Graph.

The Recall-Precision Graph is created using the 11 cutoff values from the Recall Level Precision Averages. Typically these graphs slope downward from left to right, enforcing the notion that as more relevant documents are retrieved (recall increases), the more nonrelevant documents are retrieved (precision decreases).

This graph is the most commonly used method for comparing systems. The plots of different runs can be superimposed on the same graph to determine which run is superior. Curves closest to the
upper right-hand corner of the graph (where recall and precision are maximized) indicate the best performance. Comparisons are best made in three different recall ranges: 0 to 0.2, 0.2 to 0.8, and 0.8 to 1. These ranges characterize high precision, middle recall, and high recall performance, respectively.

II. Average Precision Histogram.

Figure 2 is a sample Average Precision Histogram.

![Average Precision Histogram](image)

Figure 2: Sample Average Precision Histogram.

The Average Precision Histogram measures the average precision of a run on each topic against the median average precision of all corresponding runs on that topic. This graph is intended to give insight into the performance of individual systems and the types of topics that they handle well.

3 Other evaluation measures

Often, a track will report other measures. This section defines the more common ones.

3.1 bpref

The bpref measure is designed for situations where relevance judgments are known to be far from complete. It was introduced in the TREC 2005 terabyte track. bpref computes a preference relation of whether judged relevant documents are retrieved ahead of judged irrelevant documents. Thus, it is based on the relative ranks of judged documents only. The bpref measure is defined as

\[
\text{bpref} = \frac{1}{R} \sum_r \left( 1 - \frac{|n \text{ ranked higher than } r|}{\min(R, N)} \right)
\]

where \( R \) is the number of judged relevant documents, \( N \) is the number of judged irrelevant documents, \( r \) is a relevant retrieved document, and \( n \) is a member of the first \( R \) irrelevant retrieved documents. Note that this definition of bpref is different from that which is commonly cited, and follows the actual implementation in trec_eval version 8.0; see the file bpref.bug in the trec_eval distribution for details.
Bpref can be thought of as the inverse of the fraction of judged irrelevant documents that are retrieved before relevant ones. Bpref and mean average precision are very highly correlated when used with complete judgments. But when judgments are incomplete, rankings of systems by bpref still correlate highly to the original ranking, whereas rankings of systems by MAP do not.

3.2 GMAP

The GMAP measure is designed for situations where you want to highlight improvements for low-performing topics. It was introduced in the TREC 2004 robust track. GMAP is the geometric mean of per-topic average precision, in contrast with MAP which is the arithmetic mean. If a run doubles the average precision for topic A from 0.02 to 0.04, while decreasing topic B from 0.4 to 0.38, the arithmetic mean is unchanged, but the geometric mean will show an improvement.

The geometric mean is defined as \( n \)-th root of the product of \( n \) values:

\[
\text{GMAP} = \sqrt[n]{\prod_{n} \text{AP}_n}
\]

where \( n \) is typically 50 for TREC tasks. Alternatively, it can be calculated as an arithmetic mean of logs:

\[
\text{GMAP} = \exp \left( \frac{1}{n} \sum_{n} \log \text{AP}_n \right)
\]

In the \texttt{trec\_eval} implementation, all individual average precision scores that are less than 0.00001 are set to 0.00001 to avoid taking logs of 0.0.